Computational Investigation of Natural Convection on a Hexagonal Cavity with a Semi-Circular Cooler Obstacle

Gazi Homayra Shirin*

Abstract: Utilizing numerical analysis, the phenomena of natural convection were examined within a two-dimensional hexagonal hollow that had a semi-circular cooler obstruction. The most essential method, the finite element method (weighted-residual method), is used to solve the governing differential equations. While the other four sidewalls of the chamber are considered to be adiabatic, the two walls are positioned at a heated T_h. In the middle of the cavity, there is a cooling obstacle that is semicircular in shape, and all of the walls are covered to prevent slippage. With Pr= 0.71 and Ra= 10^3, 10^4,10^5, and 10^6, the research was completed. Streamlines, isotherms, fields of velocity and temperature, and the local Nusselt number are all shown in the results.

Keywords: Natural Convection, Hartmann Number, Rayleigh Number, Hexagonal cavity, Semi-circular cooler.

Introduction

Efficient magnetic field-induced convection heat transfers are crucial to contemporary technology and numerous industrial domains such as heat exchangers, thermal insulation, electronics cooling, solar technologies,crystal growing,food processing, manufacturing of petroleum and nuclear reactor.Therefore,it is vital to investigateand reproducethis phenomenon.Various experimental and numerical approaches have been devised to study cavities both with and without obstacle, and these methods find practical use in engineering and industry. Prior research mostly focused on the mechanics of distinct flow systems in distinct cavities.

Osterrach [1-3] and Hoogendoorn [4] performed an extensive literature review and investigation within a rectangular hollow.The energy and momentum equations subject to boundary conditions can be developed simultaneously using finite element analysis, which was presented by Reddy [5]. The solutions to differential equations with constant coefficients that are found at the nodes are precise.Natural convection inside a sinusoidal surrounding with different cylinder shapes was analyzed by Hussein et al. [6] using entropy generation. As the Rayleigh

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number increases, their results demonstrate that the local Beja drops and the entropy generations due to heat transfer, fluid friction, and total entropy generation all increase. Using the Lattice Boltzmann Method, Arun et al. [7] examine issues related to natural convection heat transport. They discovered that the lattice Boltzmann method could be applied in the computational sector and that the natural convection problem was important.

Magneto hydrodynamic natural convection in a square cavity with a semicircular heated obstruction was investigated by Muhammad Hussain et al. [8]. Bhuiyan et al. [9]. They studied the flow field in a square cavity with a semi-circular heated block and a uniform magnetic field, taking into account the effects of Rayleigh and Hartmann numbers.The study conducted by Alam et al. [10] focused on the numerical simulation of natural convection within a rectangular lacuna, incorporating triangles of varying orientations in the presence of a magnetic field. The parameters outlined also have a significant impact on the average Nusselt number at the hot wall and the average temperature of the fluid within the enclosure.

Runa et al. [11] examined a two-dimensional semicircular top enclosure containing a triangle obstruction within a rectangular cavity. The results indicate that the rate of heat transfer increases with the buoyant force due to a rise in the Rayleigh number and decreases with an increase in the Hartmann number.The impact of Hartmann Number on the free convective flow of MHD fluid in a square cavity with a heated cone of different orientation was examined by Saika et al. [12]. They stated that heat transmission would only occur by conduction if a magnetic field was strong enough to completely halt fluid flow. Researchers Hakan and Khaled et al. [13] examined mixed convection with mixed heat and density in a cavity with a lid and a corner heater.The study conducted by Oztop and Dagtekin[14] examined natural convection heat transfer within a square cavity featuring a heated plate installed both vertically and horizontally. The study examined the impact of the position and aspect ratio of the heated plate on heat transfer and fluid flow. It was observed that the mean Nusselt numbers at both vertical and horizontal locations increased with an increase in the Rayleigh number.

Natural convection flows inside a square cavity were studied by Basak et al. [15] in relation to thermal boundary conditions.In a slanted rectangular enclosure with neighboring walls heated and cooled, Mehmet and Elif [16] investigated natural convection flow in the presence of a magnetic field. It was noted that the flow and temperature fields are significantly affected by the magnetic field's strength and direction, as well as the enclosure's orientation and aspect ratio.In a square cavity that was heated from below and cooled from other walls, Jani et al. [17] used numerical methods to study magneto hydrodynamic free convection. Utilizing the finite volume method, they demonstrated that the transfer mechanisms, temperature distribution, and flow properties within the cavity were highly influenced by the magnetic field strength and the Rayleigh number. Mahmoodi et al. [18] examined numerically magneto-hydrodynamic free convection heat transport in a square enclosure heated from side and cooled from the ceiling.The efficiency of pulse tube refrigerators was studied statistically by Rout et al. [19].

It would appear, on the basis of the assessment of the relevant literature, that there is no work that has been reported on the free convection flow in a cavity that is hexagonal in shape and contains a semicircular obstacle. Themain objective of the present study is to examine the effect of Reynolds number on fluid flow and heat transfer in a hexagonal shaped cavity with semicircular obstacle.

Physical Configuration

The schematic of the problem, together with its boundary condition, is illustrated in Figure 1. The physical model consists of a two-dimensional hexagonal shaped cavity with semi-circular block is consider for the simulation purpose. The height and thewidth of the cavity are denoted by L. The twoside walls are kept at heated (T_h) and the semicircular block is kept atcold (T_c) under all situations $T_h > T_c$ condition is maintained.The remaining walls are adiabatic. The gravitational acceleration g, acts in the vertically downward direction.The fluid ispermeated by a uniform magnetic field B_0 which is applied parallel to the direction ofthe flow.

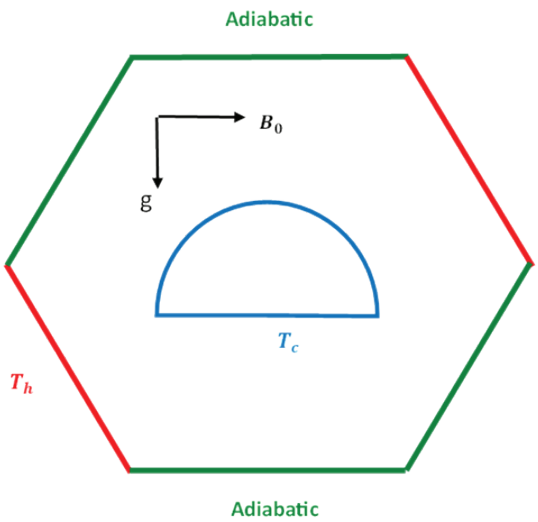

Figure 1. Schematic representation of the physical arrangement along with the boundary conditions addressed in this research.

Mathematical Formulation

The dimensionless governing equations for continuity, momentum, and energy for a 2D, steady, laminar, and incompressible flow with the Boussinesq approximation can be expressed as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad 1$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \qquad 2$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + RaPr\theta \qquad 3$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \qquad 4$$

Where,

$$X = \frac{x}{L}, \; Y = \frac{y}{L}, \; U = \frac{u}{V_0}, \; V = \frac{v}{V_0}, \; P = \frac{p}{U_0^2}, \; \theta = \frac{T - T_c}{T_h - T_c}, \; \mathrm{Ra} = \frac{g\beta\,(T_h - T_c)L^3}{v\alpha}, \; Pr = \frac{v}{\alpha}$$

The local Nusselt number (Nu) helps us understand how much heat is being transferred in the system from the active walls. In this manner, the average quantity for HT rate is evaluated by the average Nusselt number (Nu), which is calculated as follows:

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$$\overline{Nu} = \frac{hL}{K} = -\frac{\partial \theta}{\partial N}L, \qquad \frac{\partial \theta}{\partial N} = -\frac{1}{L}\sqrt{\left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial Y}\right)^2}$$

$$Nu = \frac{1}{S}\int_0^S \overline{Nu}\,dn$$

The non-dimensional boundary conditions under consideration can be written as: $U = 0$, $V = 0$, $\theta = 0$;
At the cooler wall; $U = 0$, $V = 0$, $\frac{\partial \theta}{\partial X} = 0$;
And $\theta = 1$ at the hot wallsand $U = 0$, $V = 0$, $\theta = 1$

Numerical Procedure

This study's numerical simulations utilized the finite element method (FEM), a widely-used computer approach for resolving intricate HT issues. A detailed representation of the geometry is achieved via FEM's discretization of the curving bottom wall cavity into a mesh of components.

By taking this route, we may build a solid basis for our investigation and accurately model HT phenomena. The governing equations (1)-(4) and their accompanying boundary conditions are solved using the FEM in this work. Using the Galerkin weighted residual technique, the controlling partial differential equations were transformed into integral equations in this simulation approach.In order to capture the quick changes in the dependent variables, this inquiry uses a non-uniform triangular mesh structure, particularly near the walls.

Result and Discussion

A numerical analysis of fluid flow and mixed convection heat transfer in a hexagonal cavity has been conducted. Throughout all simulation sets, the fluid inside the cavity is regarded as air, with a Prandtl number of 0.71.Following portion, we have finished estimating Ra =10^3, 10^4,10^5 and10^6 and Ha = 0. Isotherms, streamlines, velocity profiles, dimensionless temperature, and the local Nusselt number are all graphically represented in the results.

Effect of Rayleigh number

In Figure2,the streamlines for $Ha = 0$ to illustrate how the Rayleigh number affects the flow field and temperature distribution.It is possible to see several cells within the cavity from the streamlines figure. To understand the impact of the Rayleigh number on the flow field and temperature distribution, it shows the results for $Ha = 0$, in the absence of a magnetic field.At $Ra = 10^3$ three cells are formed with three elliptic- shaped eyes on upper right, middle-left and bottom-left of the cavity around the block shown in Figure. 2(a). The cells on the bottom left spin counter clockwise, while the ones on the top right spin clockwise. The stream function has symmetrical values about the vertical central axis, corresponding to this symmetry of the semi-circular coldblock. Results for larger Rayleigh numbers are virtually identical to those in Figure 2(a), with the exception that Figures 2(b)-(d) show an increase in flow power.

Conduction-dominant heat transmission is evident from the isotherms in Figures 3(a) to 3(d) at Ra = 10^3, Ra =10^4 , Ra =10^5 and Ra = 10^6. It was noted that there was a minor movement of isothermal lines away from the partially heated surface and toward thesemicircular block. Formation of thermal boundary layers canbe found and increases from the isotherms for Ra = 10^5 and Ra = 10^6 are shown in Figure3(c) and Figure 3(d) which means increasing heat transfer through convection.

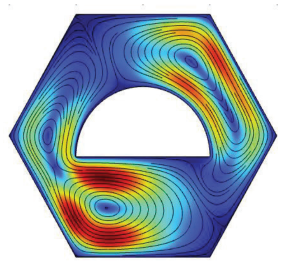
Velocity and Temperature profiles

The y-component of velocity along the line parallel to the x axis (y = 0.15) ispresented.Figure4(a) illustrates the impact of varying Rayleigh numbers (Ra) with *Pr* = 0.71.It clearly indicates that the velocity exhibits a more significant variation with lower Rayleigh numbers. The velocity reaches its maximum at the position $x<0.2$ when Ha = 0.

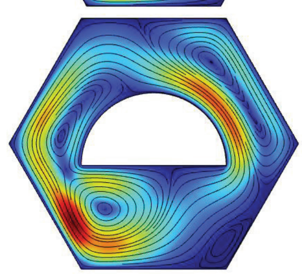
Figure 4(b) illustrates the varying Rayleigh numbers on the y-component of temperature.Figure 4(a)-(b) presents the four profiles corresponding to the values of Ra =10^3,10^4,10^5 and 10^6.Varying Rayleigh numbers provide distinct curves. The maximum form curve is attained at the highest Ra number which is 10^6.

Nusselt number

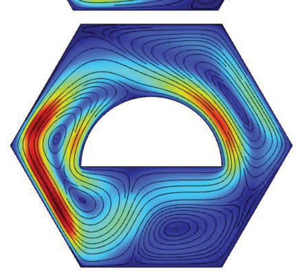
A representation of the local Nusselt number at various Rayleigh numbers (Ra) with Pr=0.71 may be found in Figure 5. When the Rayleigh number is at its highest, we reach the maximum shape curve, and when it is at its lowest, we reach the minimum value.



(a)Ra=10^3



(b)Ra=10^4



(c)Ra=10^5

(d)Ra=10^6

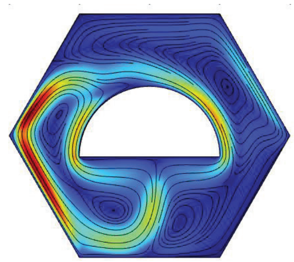


Figure 2. Streamlines for (a) Ra = 10^3, (b) Ra = 10^4, (c) Ra = 10^5, (d) Ra = 10^6 while *Pr* = 0.71

(a)Ra=10^6

(b)Ra=10^6

(c)Ra=10^6

(d)Ra=10^6

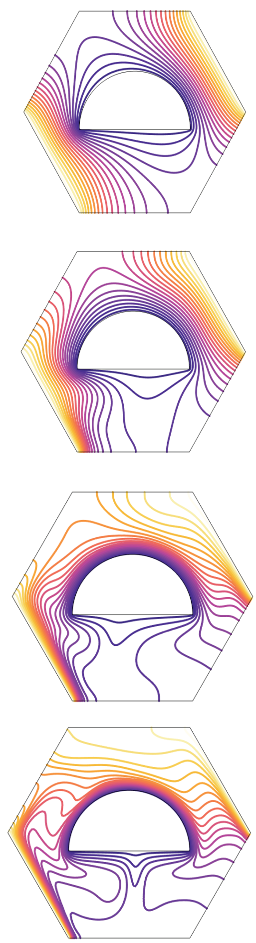


Figure 3. Isotherm for (a) Ra = 10^3, (b) Ra = 10^4, (c) Ra = 10^5, (d) Ra = 10^6 while *Pr* = 0.71

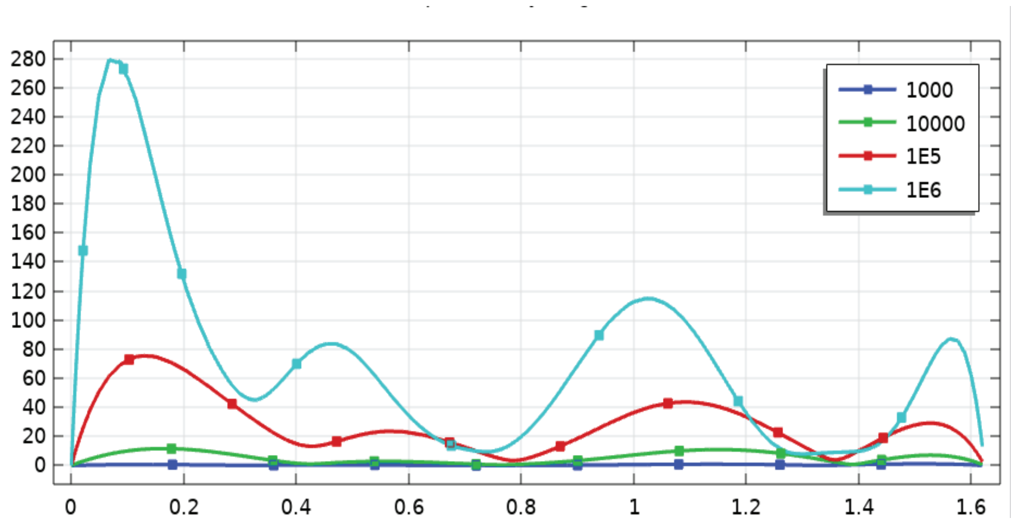


Figure 4(a). Variation of velocity magnitude profiles at different Rayleigh number with Pr = 0.71

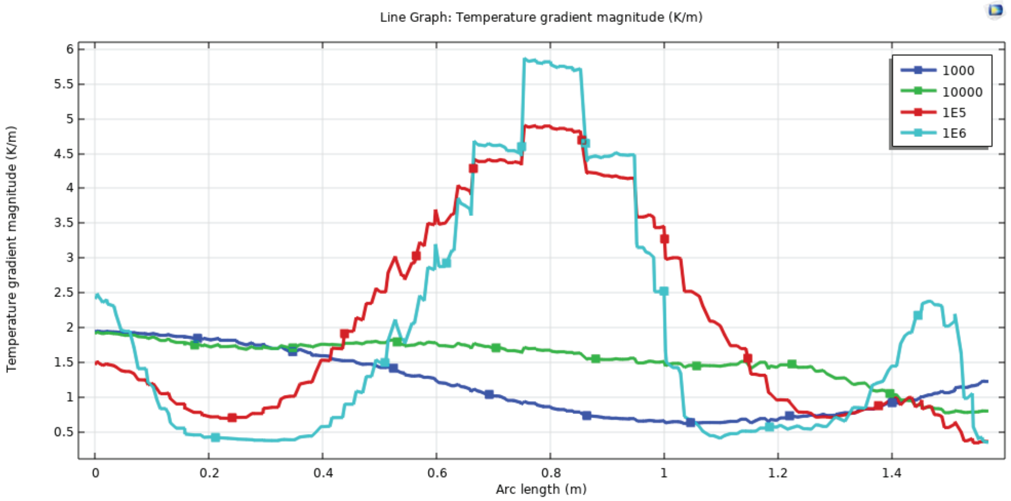


Figure 4(b). Variation of temperature profiles at different Rayleigh number with Pr = 0.71

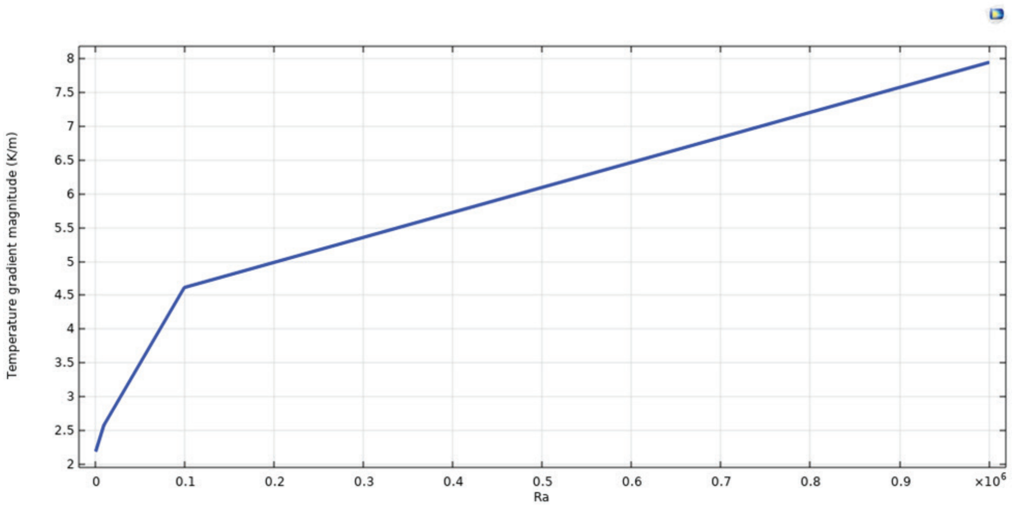


Figure 5. Variation of Nusselt number with Ra.

Conclusion

The heat transfer and magneto-hydrodynamic natural convection fluid flow in a cavity of a hexagonal form with a semi-circular block are examined in a numerical analysis. Air is the fluid under consideration.The Galerkin weighted residual method of finite element formulation was used to solve the governing equations of mass, momentum, and energy. The various Rayleigh numbers were found to have quite satisfactory agreements with one another.Three or more counter rotating eddies were generated inside the cavity independent of the Rayleigh, Hartmann, or Prandtl numbers. This was the case for all of the examples that were taken into consideration. The results that were obtained demonstrated that the processes for heat transfer, the distribution of temperature, and the flow characteristics within the cavity were highly dependent on the strength of the magnetic field as well as the Rayleigh number. Here are some findings that can be derived from the current investigation: Raising the buoyant force causes an increase in the Rayleigh number, which in turn increases the rate of heat transfer. In addition, the strength of the magnetic field is an important element to adjust when dealing with heat transfer and fluid flow.

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