Numerical Analysis of Natural Convective Heat Transfer of Cu-water Nanofluid in Square Cavity with a Circular Disk

Fariza Tul Koabra*

Abstract : The convective heat transfer enhancement of Cu-water nanofluid in a differentially heated square cavity with a circular disk was investigated. A finite element model consisting of Navier-Stokes, continuity and energy equations was developed. Thermophysical properties of Cu-water nanofluid were taken from literature data. The model was validated against established solutions for natural convection of air and nanofluid inside a differentially heated square cavity. Average Nusselt numbers were calculated for various flow conditions obtained by varying solid volume fraction of the nanoparticles and different Rayleigh numbers.

Keywords: Nanofluid, Natural convection, Finite Element Method, Nusselt number

1. Introduction

Natural convection fluid flow and heat transfer are encountered in a number of engineering and industrial applications such as cooling of electronic equipment, solar energy, and geophysics. There are a number of very recent studies, using conventional numerical methods, on the free convection heat transfer in cavities filled with nanofluids. Nanofluid has been under extensive analysis in the last decade due to its potential in heat transfer enhancement related application. Nanometer sized particles finely dispersed in a base fluid is known as nanofluid which exhibits higher heat transfer coefficient than that of base fluid[1]. This phenomenon profoundly opens up doors for application of nanofluid in micro-cooling, nano-scale drug delivery and energy conversion. An extensive review on recent works on nanofluid can be found very helpful for the researchers working in this subject [2]. Both numerical and analytical many experimental studies[3] have been conducted as well as many theoretical studies[4]. Abu-nada and Oztop[5] conducted a numerical investigation on the effect of inclination angle of a square cavity on the free convection of the Cu-water nanofluid inside it. Numerical results for free convectionin a square cavity cooled from its two vertical and the top hori zontal walls and

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heated by a constant flux heaters on its horizontal bottom wall filled with a nanofluid were reported by Aminossadati and Ghasemi [6]. In the present paper, a finite element model has been developed to analyze the heat transfer enhancement of Cu-water nanofluid in differentially heated square cavity with a circular disk. The effect of heat transfer enhancement of Cu-water nanofluid has also been addressed. The model has been validated against established results for natural convection of air and nanofluid in a differentially heated square cavity.

2. Theoretical Formulation

2.1 Properties of nanofluid

A schematic view of a square cavity with a circular disk considered in the present study is shown in Fig. 1, side of the square cavity and diameter of the circular disk are denoted by h and d respectively. The problem is formulated in two-dimensional Cartesian coordinate system. The cavity is filled with Cu-water nanofluids which is considered to be Newtonian, laminar and incompressible. Nanoparticles and base fluid are in thermal equilibrium and there is no slip between them.

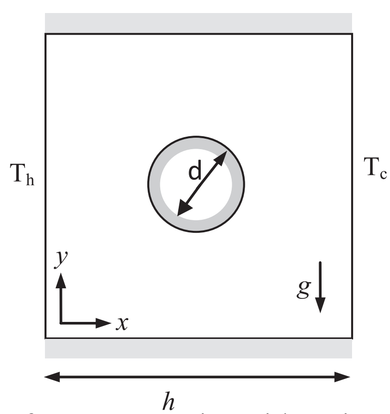

Fig.1. Schematic view of a square cavity with a circular disk The properties of nanofluid are obtained as following:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s$$

$$C_{p,nf} = \left[(1-\varphi)\rho_f C_{p,f} + \varphi\rho_s C_{p,s}\right]/\rho_{nf} \quad (1)$$

$$\rho_{nf}\beta_{nf} = (1-\varphi)\rho_f\beta_f + \varphi\rho_s\beta_s$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$$

where φ is the solid volume fraction, ρ is the density, Cp is the specific heat capacity, β is the thermal expansion coefficient, α is the thermal diffusivity, and k is the thermal conductivity. Here, subscript s, f and nf indicate physical properties of solid nanoparticle, base fluid, and nanofluid, respectively. The effective dynamic viscosity, μnf, of the Cu-water nanofluid is calculated from according to the Brinkman model using fluid viscosity μf [9]

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}} \tag{2}$$

The effective thermal conductivity of the nanofluid is determined using the Maxwell model [10]

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{(k_f + 2k_s) + \varphi(k_f - k_s)} \tag{3}$$

Physical property of the Cu-water nanofluid is determined following in the Table 1.

Table 1. Thermo-physical properties of water and Cu nanoparticle

Physical properties	Water	Cu
C (J/kg K)	4179	385
ρ (kg/m^3)	997.1	8933
k (W/m K)	0.613	400
β (K^{-1})	21x 10^{-5}	1.67 x 10^{-5}
α (m^2 s^{-1})	1.471 x 10^{-7}	-

2.2 Theoretical description:

The set of equations that governs the thermo-fluid flow is given as:

$$\begin{gathered}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\
u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\
u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{(\rho_{nf}\beta_{nf})}{\rho_{nf}} g(T - T_c) \\
u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)
\end{gathered} \tag{4}$$

where u and v are the fluid velocity in x-direction and y-direction respectively, p is the pressure, and T is the temperature.

Governing equations of the convective heat transfer are non-dimensionalized with the following dimensionless parameters defined as:

$$X = \frac{x}{h}; \quad Y = \frac{y}{h}; \quad U = \frac{uh}{\alpha_f}; \quad V = \frac{vh}{\alpha_f}; \quad P = \frac{ph^2}{\rho_{nf}{\alpha_f}^2}; \ \theta = \frac{T - T_c}{T_h - T_c}$$

The dimensionless forms of the governing equations are:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}RaPr\theta \tag{4}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right)$$

where U and V are the scaled fluid velocity in X-direction and Y-direction, respectively, P is the scaled pressure, θ is the scaled temperature. Rayleigh number Ra and Prandtl number Pr are defined as

$$Ra = \frac{g\beta_f(T - T_c)h^3}{\alpha_f v_f}; \quad Pr = \frac{v_f}{\alpha_f}$$

In order to evaluate the heat transfer enhancement in the cavity, the local Nusselt number on the walls is defined as:

$$Nu_l = -\frac{k_{nf}}{k_f}\frac{\partial \theta}{\partial n}\bigg|_{wall}$$

Average Nusselt number along the hot walls of the cavity is considered to evaluate the overall heat transfer rate and is defined as:

$$Nu_{avg} = \int_0^1 Nu_l dY|_{X=0}$$

3. Results and Discussion

3.1 Model Validation

The model is solved by using commercial finite element package COMSOL Multiphysics. It has been validated against benchmark solutions obtained in the literature as shown in Table 2. Natural convection of air inside a square cavity whose two sides are set to differential temperatures while keeping the top and bottom surfaces at adiabatic condition is a classic case for validation. Average Nusselt number for the hot wall calculated from the present model is compared with the data available in the literature and found very accurate for various high Rayleigh numbers.

Table 2. Average Nusselt number for the hot wall of the air filled square cavity obtained by various studies are shown and compared with present study for different Rayleigh numbers

Rayleigh number (Ra)	Present study	Vahl Davis (1983) [11]	Fusegi et al.(1991) [12]	Comini et al.(1995) [13]	Khanafer et al.(2003) [3]	Bilgen (2005) [14]
10^4	2.2448	2.243	2.302	-	2.245	2.245
10^5	4.5216	4.519	4.646	4.503	4.522	4.521
10^6	8.8262	8.799	9.012	8.825	8.826	8.800
10^7	16.5301	-	16.543	16.533	-	16.629

A second degree of validation has been done for Cu-water nanofluid. Calculated average Nusselt number of the present study was compared against that of a Cu-water nanofluid filled square cavity for various Grashof number and found satisfactory.

Table 3. Comparison of average Nusselt number calculated on hot wall in Cu-water nanofluid filled square cavity with Khanafer et al.(2003):

Grashof number (Gr)	Present study	Khanefer et al (2003) [3]
10^3	2.5662	2.835
10^4	5.4050	5.895
10^5	10.6669	11.245

3.2 Adiabatic circular disk

The first problem was defined as differentially heated square cavity with an adiabatic circular disk at the centre of the cavity. The ratio of diameter of the disk *(d)* to sides of the square cavity *(h)* is taken as $\lambda=d/h=0.2$. Thermal boundary conditions are taken as $T=T_h$ for the hot left wall, $T=T_c$ for the cold right wall and adiabatic condition $\partial T/\partial n=0$ for the top, bottom and disk walls. No-slip condition is imposed on all surfaces.

Using the dimensionless parameters in Eq. (4) the following boundary conditions are obtained.

On the left wall: $U=V=0; \quad \theta=1$

On the right wall: $U=V=0; \quad \theta=0$

On the top and bottom walls and disk: $U=V=0; \quad \partial\theta/\partial n=0$

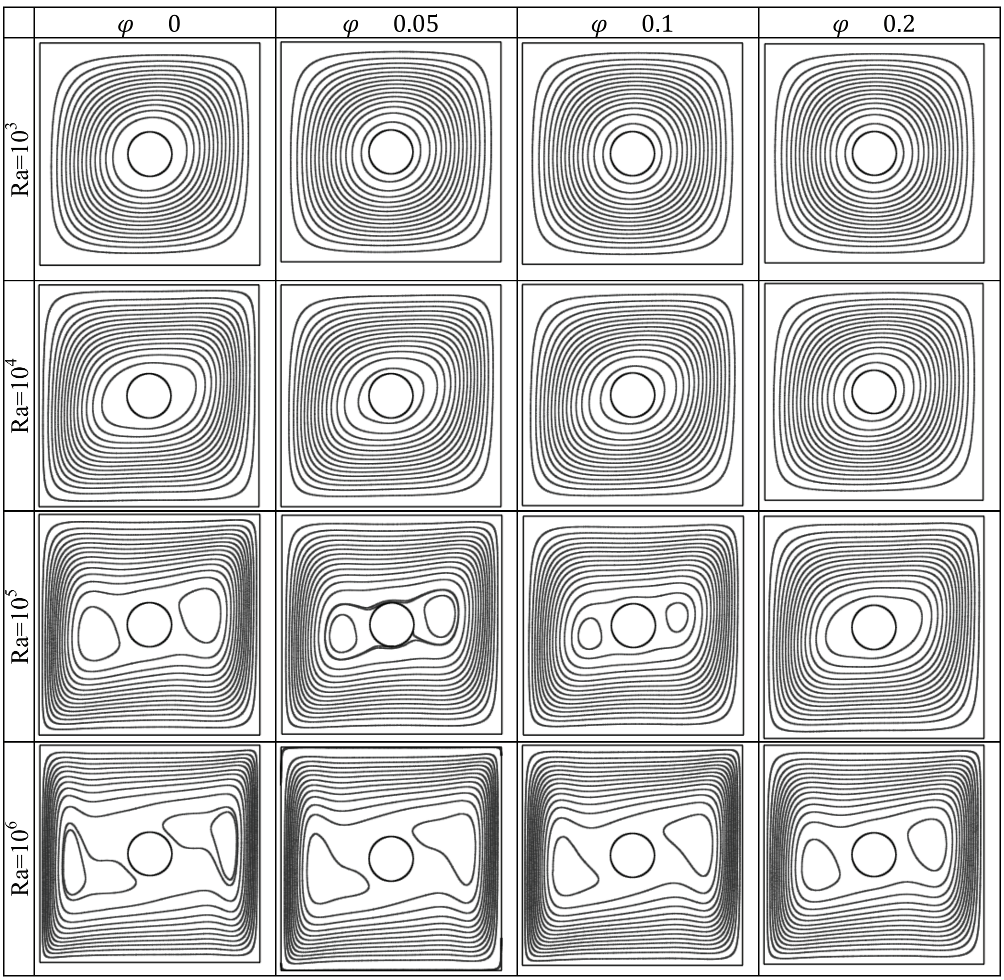

Fig.2. Streamlines for Cu–water nanofluid in a differentially heated square cavity with an adiabatic circular disk for different volume fractions and Rayleigh numbers

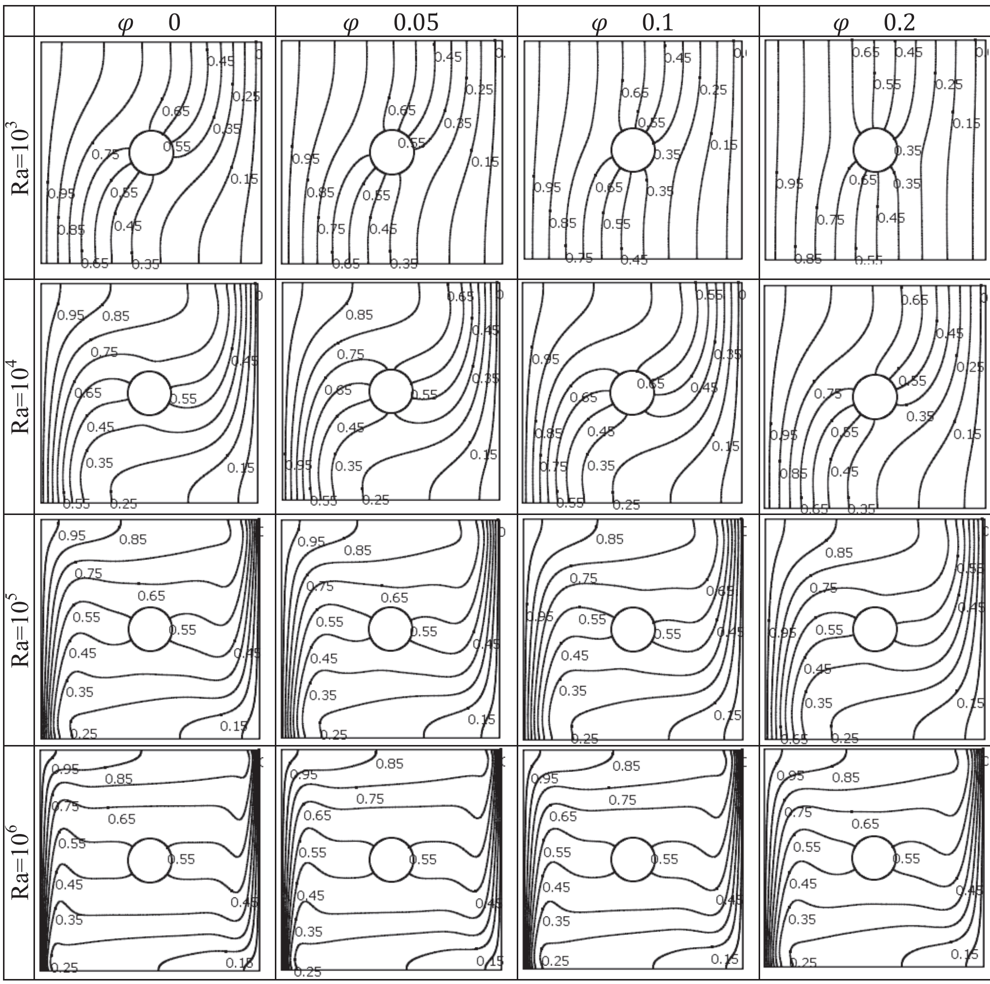

Fig.3. Isotherms for Cu–water nanofluid in a differentially heated square cavity with an adiabatic circular disk for different volume fractions and Rayleigh numbers

3.3 Cold circular disk

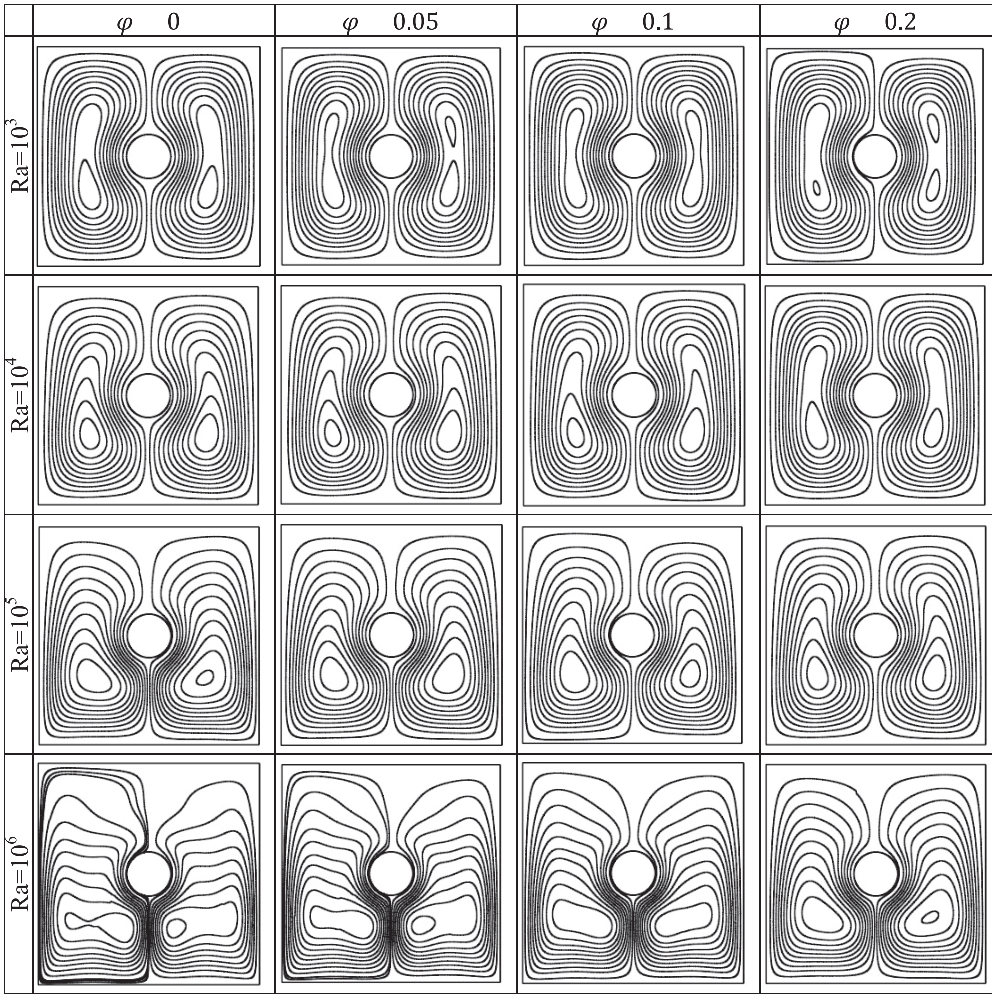

Fig.4. Streamlines for Cu–water nanofluid in a square cavity with a cold circular disk at different volume fractions and Rayleigh numbers

Eq. (4) the following boundary conditions are obtained for cold circular disk.

On the disk wall: $U=V=0; \quad \theta=0$

On the left and right walls: $U=V=0; \quad \theta=1$

On the top and bottom walls: $U=V=0; \quad \partial\theta/\partial n=0$

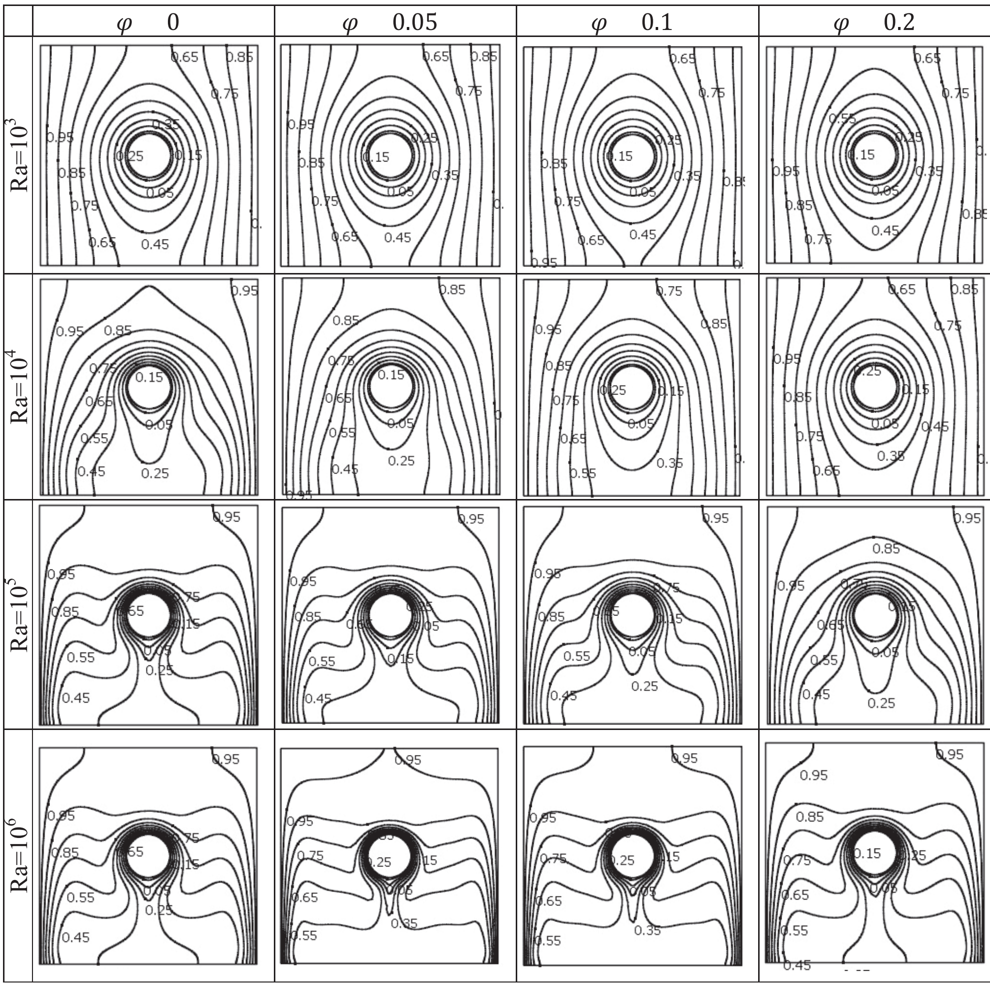

Fig.5. Isotherms for Cu–water nanofluid in a square cavity with a cold circular disk for different volume fractions and Rayleigh numbers

3.4 Heated circular disk

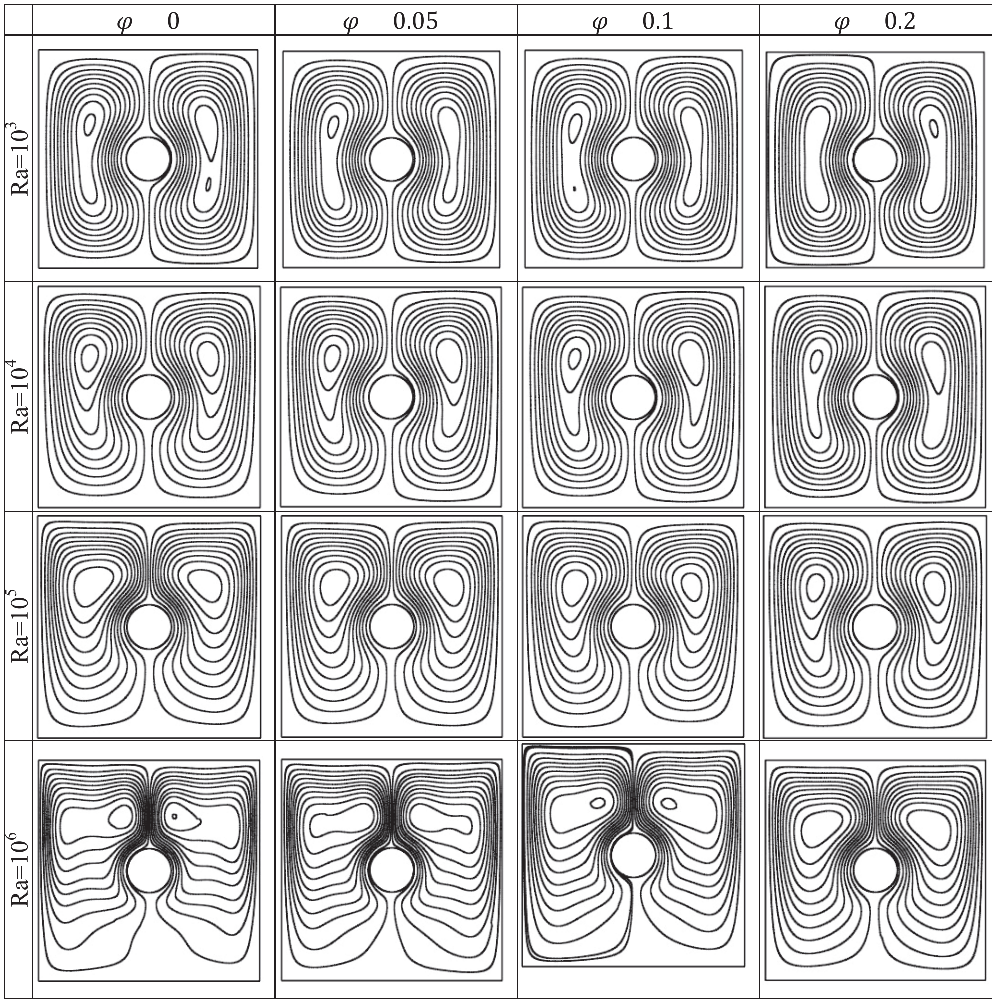

Fig.6. Streamlines for Cu–water nanofluid in a square cavity with a heated circular disk for different volume fractions and Rayleigh numbers

Eq. (4) the following boundary conditions are obtained for heated circular disk.

On the disk wall: $U=V=0; \quad \theta=1$

On the left and right walls: $U=V=0; \quad \theta=0$

On the top and bottom walls: $U=V=0; \quad \partial\theta/\partial n=0$

In the present investigation, the influences of Rayleigh number Ra and solid volume fraction of nanoparticles φ on the streamlines and isotherms were assumed while Prandtl number is kept constant at $(Pr = 6.2)$. The values of Rayleigh numbers and solid volume fraction of nanoparticles considered are $Ra = 10^3, 10^4, 10^5, 10^6$ and $\varphi = 0, 0.05, 0.1, 0.15, 0.2$. Beside these, the average Nusselt numbers in the enclosure have been calculated in the Fig. 2, which is related to streamlines and temperature lines of the Cu-water nanofluid at lower values of Ra isotherms formed in the cavity and especially in the vicinity of the lids which shows the dominance of conduction heat transfer. The effects of Rayleigh number Ra upon the streamline patterns have been presented in Fig.2. while $\mathrm{Pr} = 6.2$ and $\mathrm{Ra} = 10^3, 10^4, 10^5, 10^6$ and $\varphi = 0, 0.05, 0.1, 0.15, 0.2$. Two primary recirculation cells are found in the streamlines at high Rayleigh number. The shapes of these vortices change from circular to triangular with the increasing of Ra . This happens as a result of getting higher buoyancy force. When volume fraction of nanoparticles increases from 0.1 to 0.2, length of the circulation cell becomes smaller

Fig.3 shows the temperature distribution for different Ra and φ . In this figure the isotherms become parallel with the increase of the suggesting inner circulation around the disk, outer circulation zone formed close to the wall.

In Fig.4 and Fig.5, the results are presented in terms of streamlines and isotherms for different choice of solid volume fractions φ and different Rayleigh numbers Ra in the square cavity containing cold circular disk. The shape of streamlines changes slightly with the increasing of the solid volume fraction because of higher concentration of nanoparticles. As the volume fraction of nanoparticles enhances from 0 to 0.2, the isotherm contours tend to get affected considerably.

In Fig.6 and Fig.7, the results are presented in terms of streamlines and isotherms for different choice of solid volume fractions φ and different Rayleigh numbers Ra in the square cavity containing heated circular disk. The flow rate exists with maximum value in the centre of the circulation. As the parameter Ra increases, the flow rates at the centers of circulation increase.

In Fig.7. for increment of Ra, the isothermal lines become more condensed near the heat source and a thermal plume formed based on the heated body due to convection are is dominated across the enclosure. Due to rising values of Ra, the temperature distributions become deformed ensuing in an augmentation in the whole heat transport process. This effect may be accredited to the control of the buoyant convection. Moreover, it is observed that raising the Rayleigh numbers cause the higher depth of the thermal boundary layer near the heated surface that point towards a steep temperature gradient and consequently, an enhancement on the whole heat transfer inside the cavity. In addition, these lines corresponding to $\varphi = 0.2$ become more bended. The isotherms are packed out about the vigorous part of the heated surface in the cavity for clear water ($\varphi = 0$). It is seen that, clear water moves more rapidly than the solid concentrated nanofluids. Rising of φ shows a deformation at the isothermal lines near the upper part of the top portion of the heated circular disk.

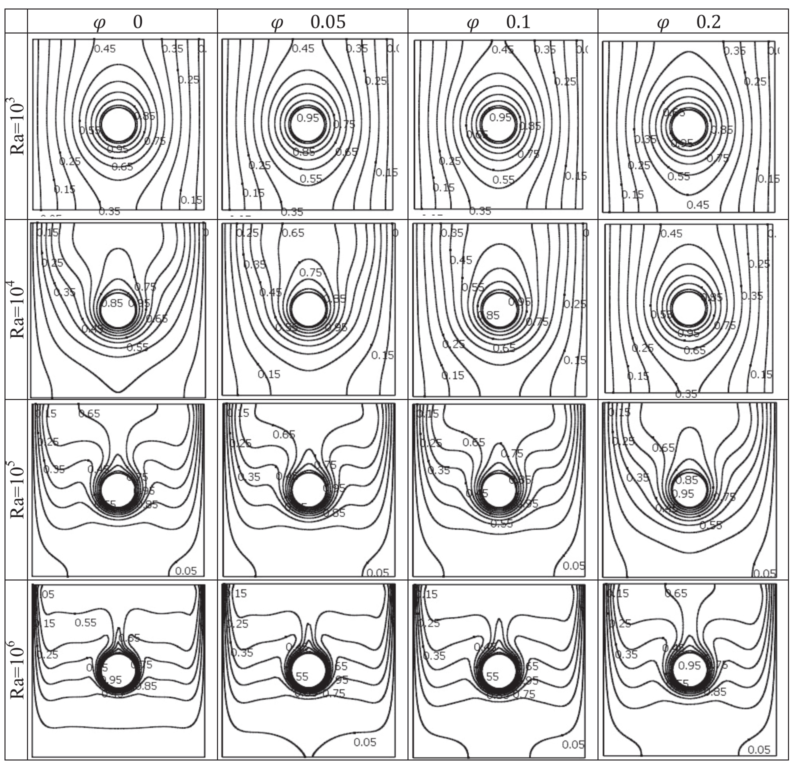

Fig.7. Isotherms for Cu–water nanofluid in a square cavity with a hot circular disk at different volume fractions and Rayleigh numbers

Figure 8(a), displays the average Nusselt number Nu_{avg} due to the Rayleigh number (Ra) as well as solid volume fraction$s(\varphi)$ effect. It is seen from figures Nu_{avg} increases for greater values of φ because nanofluid has greater thermal conductivity in comparison to pure water.

Figure 8(b), presents the variation of average Nusselt number Nu_{avg} with volume fraction (φ) using different values of Rayleigh number. The figure shows that the heat transfer increases almost monotonically with increasing the volume fraction for all Rayleigh numbers .

Figure 8(c), displays the average Nusselt number Nu_{avg} due to the Rayleigh numbers (Ra) as well as solid volume fraction$s(\varphi)$ effect. It is seen from figures that Nu_{avg} enhances sharply upto $Ra = 10^4$ and beyond this region it rises gradually.

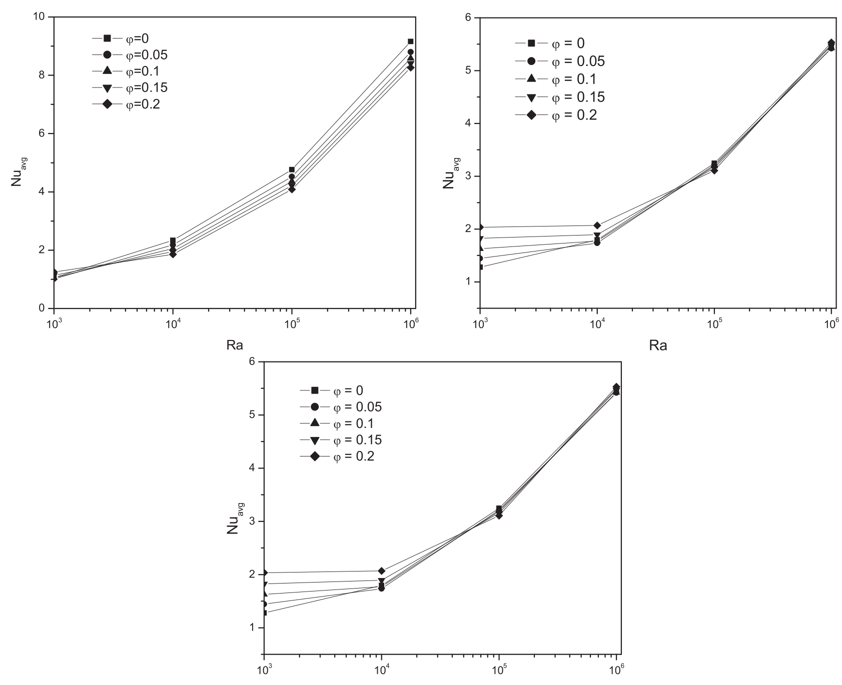

Fig.8 Average Nusselt number for different volume fraction of (a) adiabatic circular disc (b) cold circular disk, and (c) heated circular disk.

4. Conclusion

A numerical investigation concerning the effects of nanoparticles concentration and natural convection parameter Ra on velocity and temperature field around an adiabatic circular disk, cold circular disk and heated circular disk placed in an enclosure filled with Cu-water nanofluids is accounted. The focal point of the present investigation is to calculate the average heat transfer rate and entropy generation of the Cu-water nanofluids with a wide choice of Rayleigh number along with solid volume fraction while Pr is fixed at 6.2. The subsequent findings can be seen from the current numerical analysis:

- Ra and φ significantly affect the configuration of the streamlines and isotherms within the square cavity containing different circular disk.
- Greater variation is observed in velocities at a particular point for the changes of Ra than φ .

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